Thermometry of arbitrary quantum systems via non-equilibrium work distributions: Application to bosons in a lattice

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CCPQ Windsor, 4th August 2015
System $\hat{H}_S$ in equilibrium $\hat{\rho}_S \propto e^{-\beta \hat{H}_S}$ at inv. temperature $\beta$

- Interact with probe e.g. qubit $(\Delta/2)\hat{\sigma}_z$
- Let probe thermalise

$$\hat{\rho}_q \propto \mathbb{1} - 2 \tanh(\beta \Delta)\hat{\sigma}_z$$

- Measure probe $\langle \hat{\sigma}_z \rangle \propto \tanh(\beta \Delta)$
- Infer $\beta$
Equilibrium probe II

Good:

- Don’t need to know $\hat{H}_S$
- Just need to know $(\Delta/2)\hat{\sigma}_z$

Bad:

- Need to precisely know and tune $\Delta \sim 1/\beta$
- Not easy at low temperatures
Non-equilibrium probe I

Popular method:

- QuProCS

For thermometry:

- Good: Not necessary that probe energy $\sim 1/\beta$

- Bad: Only applicable to a few simple systems
Non-equilibrium probe II

What we need:

- Generic temperature-dependence in non-equilibrium probe dynamics

Where do we get this:

- Relationship between non-equilibrium work-distributions
Non-equilibrium work distributions

Consider a quench $Q$:

- System Hamiltonian $\hat{H}(\lambda) = \hat{H}_S + \lambda \hat{V}$
- System begins in equilibrium
- Parameter $\lambda$ changed over time $\tau$
- Measure the work distribution $P_Q(W)$

Consider a forward and backwards quench

- $F$ from $\lambda_i$ to $\lambda_f$
- $B$ from $\lambda_f$ to $\lambda_i$


$$\ln \left\{ \frac{P_F(W)}{P_B(-W)} \right\} = \beta(W - \Delta F)$$

with free energy difference $\Delta F = F_f - F_i$
Qubit interferometry I

How to measure work distribution $P_Q(W)$?


- Add qubit probe $\hat{H}_T(t) = (\Delta/2)\hat{\sigma}_z + \hat{H}_S + \hat{H}_I(t)$

- Each state interacts with different interaction strength

  $$\hat{H}_I(t) = (g_\downarrow(t)|\downarrow\rangle\langle\downarrow| + g_\uparrow(t)|\uparrow\rangle\langle\uparrow|) \otimes \hat{V}$$

- Start at $t = 0$

  $$\hat{\rho} = |+\rangle\langle+| \otimes \hat{\rho}_S(\lambda_Q(0))$$

  with the qubit in some superposition $|+\rangle = (|\downarrow\rangle + |\uparrow\rangle)/\sqrt{2}$
Qubit interferometry II

- Quenched with a delay $u$

\[ \lambda(\tau) \]

\[ g_{\downarrow}(t) \]
\[ g_{\uparrow}(t) \]

\[ \lambda(0) \]

\[ t \]
Qubit interferometry III

- Qubit ends up in state

\[ \hat{\rho}_q = \frac{1}{2} \left( \mathbb{1} + c_x \hat{\sigma}_x + i c_y \hat{\sigma}_y \right) \]

\[ c_x + i c_y \propto \chi_Q(u) \]

with decoherence (characteristic) function \( \chi_Q(u) \)

- Measure it

\[ \langle \hat{\sigma}_x \rangle + i \langle \hat{\sigma}_y \rangle \propto \chi_Q(u) \]

- Find work distribution

\[ P_Q(W) = (2\pi)^{-1} \int du e^{-iWu} \chi_Q(u) \]
Ideal thermometry

What don’t we need:
  ▶ System Hamiltonian $\hat{H}_S$
  ▶ Coupling $\hat{V}$
  ▶ Tuned qubit $\Delta \sim 1/\beta$

What do we need:
  ▶ Both qubit states couple to the same (potentially unknown) $\hat{V}$
  ▶ The quenches match
Inferring $\beta$ I

- Estimate $\chi_Q(u_j)$ using $N_{\text{meas}}$ measurements, $N_{\text{steps}}$ time point, up to $T$
Inferring $\beta$ II

Estimate

$$p_Q(W) = \frac{T}{2\pi N_{\text{steps}}} \left( 1 + 2\Re \sum_{j=1}^{N_{\text{steps}}} e^{-iWu_j} \chi Q(u_j) \right)$$

(b) $p_F(W)J$

(b) $p_B(W)J$

Estimate $\pm \sigma$
Inferring $\beta$ III

- Use Crooks’ relation $\ln\{p_F(W)/p_B(-W)\} = \beta(W - \Delta F)$ and Bayesian analysis

$$
\mathcal{P}(\beta, \Delta F|O) = \frac{\mathcal{P}(O|\beta, \Delta F)\mathcal{P}(\beta, \Delta F)}{\int d\beta d(\Delta F)\mathcal{P}(O|\beta, \Delta F)\mathcal{P}(\beta, \Delta F)}
$$

to classify state of knowledge given observations $O$
Example system: cold atoms, BHM

General:
- System — cold atomic gas
- Qubit — two internal states of an impurity atom of a different species, strongly localised
- Coupling — both states, $\hat{V} = \int d\mathbf{r} n_q(\mathbf{r})\hat{\Psi}^\dagger(\mathbf{r})\hat{\Psi}(\mathbf{r})$

Specific:
- Bose Hubbard model

$$\hat{H}_S = -J \sum_{\langle jj' \rangle} \hat{a}_j^\dagger \hat{a}_{j'} + \sum_{j=1}^{M} \left( \frac{U}{2} \hat{a}_j^\dagger \hat{a}_j^\dagger \hat{a}_j \hat{a}_j - \mu \hat{a}_j^\dagger \hat{a}_j \right)$$

- On-site interaction

$$\hat{V} = \eta \hat{a}_c^\dagger \hat{a}_c$$
Results — superfluid I

\[\Re\{\chi_F(u)\}\]
\[\Im\{\chi_F(u)\}\]
\[\pm|\chi_F(u)|\]

Estimate ± \(\sigma\)

\[\beta J = 1\]
\[nU/J = 0.1\]
\[\lambda_i \eta/J, \lambda_f \eta/J = 0, 0.5\]
\[\tau J, T J = 1, 9\]
\[N_{\text{meas}} = 500\]
\[N_{\text{steps}} = 200\]
Results — superfluid II

Params as before, except $N_{\text{steps}} = 1000$. 
Results — stronger interactions

\[ \beta J = 1 \]
\[ nU/J = 4 \]
\[ \lambda_i \eta/J, \lambda_f \eta/J = 0, 2 \]
\[ \tau J, TJ = 0.1, 10 \]
\[ N_{\text{meas}} = 500 \]
\[ N_{\text{steps}} = 200 \]
Summary and conclusions

Key points:
- Low-temperature
- Generic
- Works for mildly-interacting Bose gas

Ideas:
- Distinguish thermal from non-thermal?
Acknowledgements

- Francesco Cosco
  (previously with Francesco Plastina @ Calabria, now Sabrina Maniscalco @ Turku)
- Mark Mitchison
  (assistance from Sarah Al-Assam and the TNT library)
- Stephen Clark
- Dieter Jaksch
Questions?

Thanks for listening